BRCM Gyankunj School

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दिनांक **:-31.05.2025**

कार्यालय आदेश

प्रिय अभिभावक,

विद्यालय में ग्रीष्मकालीन अवकाश
दिनांक 01 जून 2025 रविवार से 30 जून 2025 दिन सोमवार तक

विद्यालय 01 जुलाई 2025 (मंगलवार) से ग्रीष्मकालीन अवकाश के उपरांत खुलेगा !

आपसे अनुरोध है कि:-

- अपने बच्चे को छुट्टियों का गृहकार्य नियमित रूप से करने के लिए प्रोत्साहित करें !
- बच्चे को अधिकाधिक समय दें, उसके साथ बातचीत करें, खेले आदि !
- सुनिश्चित करें कि बच्चा मोबाइल फोन का प्रयोग बहुत कम करें ! मोबाइल फोन का प्रयोग अति आवश्यक होने पर ही करने दें !
- ग्रीष्मकालीन अवकाश के दौरान विद्यालय कार्यालय खुला रहेगा!(समय 09:00am से 01:00pm)
- यदि आपके बच्चे की अब तक की फीस जमा नहीं की गई है तो कृपया शीघ्र जमा करें, अन्यथा विलंब शुल्क लगेगा !
- आप फीस ऑनलाइन भी जमा कर सकते हैं, ऑनलाइन फीस जमा कराने पर निम्न का प्रयोग करें !

Noted:- फीस जमा करने पर (बच्चे का नाम और आईडी नंबर के साथ) इस नंबर पर स्क्रीनशॉट व्हाट्सएप करें ! **+91 9991500255 (Accounts Officer)**

Account Holder Name – BRCM Gyankunj School
Account Number. 7065928943
IFSC Code:- IDIB000B547
Bank Name :- Indian Bank - Bahal

आपका सहयोग ही हमारी सफलता का आधार है !
धन्यवाद !

प्राचार्य

BRCM GYANKUNJ SCHOOL

BAHAL-127028, BHIWANI

SUMMER HOLIDAY HOME WORK- 2025

CLASS: XII (Commerce) and (Humanities)

English:- **Do all the written work in English Class Work copy**

Do your work in neat writing and write the date and day

- Read and make summary of 200 words of the chapters Indigo and Poets and Pancakes.
- Learn all the answers of the poems and chapters done in the class.
- Write one notice on Appeal and Tour and Trek .
- Write a letter to the Editor of the national daily expressing your concern about Erratic Power cut.
- Write job applications for the post of Librarian and an accountant.
- Write an article in 150 on any topic of your choice.
- Write one formal and informal invitations on any event of your choice.
- PROJECT WORK:
- Do project work as discussed in the class.
- Make a project file on any topic of your choice from English syllabus.
- Write speech in 200 words on any topic of your choice.

Submit your homework on the first day of the school reopen after summer holidays.

Hindi:-

- पठित पाठों एवं कविताओं को पढ़कर उनका सारांश अपने शब्दों में लिखे ,एवं पाठ से संबंधित पांच पांच बहुविकल्पीय प्रश्न बनाकर लिखिए।
- उपरोक्त सभी कार्य विद्यालय की प्रोजेक्ट फाइल में करना है

Accountancy:-

- Prepare Board Practical file on topic
  Cash flow statement Accounting Ratio
- Solve all numerical questions related to following chapters.
  Issue of Share Issue of Debentures Financial tools Accounting Ratio

Economic:-

- Do exercise questions of chapter 1,2and 3 in Indian Economic Development and learn them.
- Do exercise questions of chapter 1,2,3 and 4 of Macroeconomics and learn them.
- Prepare a project file from any one topic of Indian economic development and one topic from macroeconomics.

Business Studies:-

- Solve all the applying and analysis based case studies in your fair notebook of chapter 1-4.
- Solve unit wise Practice Paper of each chapter 1-4.
- Prepare a practical file with any two topics from book.

General instruction to prepare the practical file

- File should be simple and brief
- Cover and Index should be neat and properly presented

EPS:-

- Write and learn all exercise questions in your notebook
- Entrepreneurship project on Business plan and market survey on any product

Political Science:-

- Book -1 Contemporary world politics
  - Write time line and 20 MCQ of Ch-1 and Ch-2
  - Write about European Union and ASEAN SAARC, INDIAN'S NUCLEAR POLICY
  - Write about agencies of UN
- Book -2 India Since Independence
  - NAM 1961 to present times.
  - Write 20 MCQ of Ch-1 and Ch-2 And Write about Indian Foreign policy

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SUMMER HOLIDAY HOME WORK- 2025

Geography:-

- Chapter 1to 9 Make 20 MCQs from each chapter.
- Practice of all the chapters Maps in India

History:-

- Project work- Select any one topic from the given below and prepare project work in history.
  - The Indus valley civilization.
  - The history and legacy of Mauryan Empire.
  - Mahabharat'- The great epic of India.
  - The history and culture of the vedic period.
  - The Bhakti Movement.
  - Global legacy of Gandhian ideas.
  - The architectural culture of the Vijayanagar empire.
  - The revolt of 1857.
  - The Philosophy of Guru Nanak Dev.
  - The vision of Kabir.
  - An insight into the Indian constitution.
- Map work- Learn and practice map syllabus of history as per CBSE.
- Prepare a flow chart on topics of your history textbook chapter 1 to 7.

Mathematics:-

BRCM GYANKUNJ

STD 12 Science Mathematics **Total Marks : 490**

Holidays Homework Assignment

➤ **Given Section consists of questions of 2 marks each.** **[100]**

1. Determine whether the relation is reflexive, symmetric and transitive:
   Relation R in the set A of human beings in a town at a particular time given by
   R = {(x, y) : x is exactly 7 cm taller than y}
2. Prove that the Greatest integer Function f : R $\rightarrow$ R, given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.
3. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.
4. Show that the Signum Function $f : R \rightarrow R$, given by
   $f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \text{ is neither one-one nor} \\ 1, \text{ if } x < 0 \end{cases}$ onto.
5. The following defines a relation on N:
   $\mathrm{x + y = 10, x, y \in N}$ Determine which of the above relations are reflexive, symmetric and transitive.
6. The following defines a relation on N:
   $\mathrm{x > y, x, y \in N}$ Determine which of the above relations are reflexive, symmetric and transitive.
7. If A = {1, 2, 3, 4} define relations on A which have properties of being:
   Symmetric but neither reflexive nor transitive.
8. Show that the function f : R → {3} → R - {2} given by $\mathrm{f(x) = \frac{x-2}{x-3}}$ is a bijection.
9. Let f be a real function given by $\mathrm{f(x) = \sqrt{x-2}}$. Find the following:
   f^2 Also, show that fof $\neq$ f^2.
10. If f : R → R, g : R → R are given by f(x) = $(x + 1)^2$ and g(x) = x^2 + 1, then write the value of fog(-3).
11. Evaluate the following:
    $\cos\left(\tan^{-1}\frac{24}{7}\right)$
12. Evaluate the following:
    $\cot^{-1}\left\{\cot\left(-\frac{8\pi}{3}\right)\right\}$
13. Find the set values of $\operatorname{cosec}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

14. If x > 1, then write the value of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in terms of $\tan^{-1}x$.

15. If $\cot\left(\cos^{-1}\frac{3}{5}+\sin^{-1}x\right)=0,$ find the values of x.

16. Solve:

$4\sin^{-1}x=\pi-\cos^{-1}x$

17. Evaluate:

$\mathrm{cosec}\left\{\cot^{-1}\left(-\frac{12}{5}\right)\right\}$

18. If -1 < x < 0, then write the value of $\sin^{-1}\left(\frac{2x}{1+cx^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

19. Solve:

$\sin^{-1}x=\frac{\pi}{6}+\cos^{-1}x$

20. Construct a 2 × 3 matrix A = [a_{ij}] whose elements a_{ij} are give by:

$a_{ij}=\frac{(i+j)^2}{2}$

21. Construct a 2 × 2 matrix A = [a_{ij}] whose elements a_{ij} are given by:

$a_{ij}=e^{2ix}\sin(xj)$

22. Find x, y, a and b if $\begin{bmatrix}3x+4y & 2 & x-2y\\ a+b & 2a-b & ^{-}1\end{bmatrix}=\begin{bmatrix}2 & 2 & 4\\ 5 & -5 & -1\end{bmatrix}$

23. Find the values of x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

24. If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix},$ show that A - A^T is a skew symmetric matrix.

25. Without expanding, show that the values of the following determinant are zero:

$\begin{vmatrix}8 & 2 & 7\\ 12 & 3 & 5\\ 16 & 4 & 3\end{vmatrix}$

26. For what value of x, the following matrix is singular?

$\begin{vmatrix}5-x & x+1\\ 2 & 4\end{vmatrix}$

27. Evaluate: $\begin{vmatrix}\cos 15^\circ & \sin 15^\circ\\ \sin 75^\circ & \cos 75^\circ\end{vmatrix}$

28. Evaluate the following determinant:

$\begin{vmatrix}a & h & g\\ h & b & f\\ g & f & c\end{vmatrix}$

29. Find the value of x, if:

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

30. If $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, find k.

31. Determine whether $f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x = 0 or not.

32. If $f(x) = \begin{cases} \frac{x}{\sin 3x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then write the value of k.

33. If the function $f(x) = \frac{\sin 10x}{x}, x \neq 0$ is continuous at x = 0, find f(0).

34. Find $\frac{dy}{dx}$, when
$x = b\sin^2\theta$ and $y = a\cos^2\theta$

35. If $-\frac{\pi}{2} < x < 0$ and $y = \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, find $\frac{dy}{dx}$.

36. Differentiate x^2 with respect to x^3.

37. Find $\frac{dy}{dx}$, when
$x = a\cos\theta$ and $y = b\sin\theta$

38. Differentiate the following with respect to x:
$\cos^{-1}(\sin\ x)$

39. If $x = f(t)$ and $y = g(t)$, then write the value of $\frac{d^2y}{dx^2}$.

40. If $y = x + e^x$, find $\frac{d^2x}{dy^2}$.

41. Find $\frac{d^2y}{dx^2}$, where $y = \log\left(\frac{x^2}{e^2}\right)$

42. The total revenue received from the sale of x units of a product is given by R(x) = $13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

43. Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2cm.

44. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm/ sec. At the instant when the radius of the circular wave is 10cm, how fast is the enclosed area increasing?

45. Write the equation of the tangent drawn to the curve $y = \sin x$ at the point (0, 0).

46. Prove that the following function are increasing on R.
$f(x) = 3x^5 + 40x^3 + 240x$

47. Show that $f(x) = x^2 - x\sin x$ is an increasing function on $\left(0, \frac{\pi}{2}\right)$.

48. Find the values of b for which the function $f(x) = \sin x - bx + c$ is a decreasing function on R.

49. Write the point where $f(x) = x \log_e x$ attains minimum value.

50. Write the minimum value of $f(x) = x + \frac{1}{x}, x > 0$.

➤ **Given Section consists of questions of 3 marks each.** **[150]**

51. Three relation R_4 is defined in set A = {a, b, c} as follows:

R_4 = {(a, b), (b, c), (c, a)} Find whether or not the relation R_4 on A is:

i. Reflexive.
ii. Symmetric.
iii. Transitive.

52. The following relation are defined on the set of real numbers.

aRb if $|a| \leq b$ Find whether these relation are reflexive, symmetric or transitive.

53. Let A = {1, 2, 3}, and let R_3 = {(1, 3), (3, 3)}. Find whether or not the relations R_3 on A is:

i. Reflexive.
ii. Symmetric.
iii. Transitive.

54. Test whether the following relations R_2 are:

i. Reflexive.
ii. Symmetric.
iii. Transitive.

R_2 on Z defined by $(a, b) \in R_2 \Leftrightarrow |a - b| \leq 5$

55. Find fog and gof if:

$f(x) = x^2 + 2$, $g(x) = 1 - \frac{1}{1-x}$

56. Find f^{-1} if it exists: f : A → B, where, A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} and $f(x) = x^2$.

57. Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = |x|

58. State with reasons whether the following functions have inverse:

g : {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

59. Solve:

$\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

60. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then write the values of x + y + z.

61. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then write the value of x + y + xy.

62. Write the following in the simplest form:

$\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

63. For the principal values, evaluate the following:

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

64. Evaluate the following:

$\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$

65. If x < 1, then write the value of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ in terms of $\tan^{-1}x$.

66. Write the value of $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)$ for x > 0.

67. Let $A=\begin{bmatrix}2 & -3\\ -7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & -4\end{bmatrix}$, verify that

$(A+B)^T=A^T+B^T$

68. For two matrices A and B, $A=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix}, B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$ verify that $(AB)^T = B^TA^T$.

69. If $x\begin{bmatrix}2\\ 3\end{bmatrix}+y\begin{bmatrix}-1\\ 1\end{bmatrix}=\begin{bmatrix}10\\ 5\end{bmatrix}$, find the value of x.

70. Find X if $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

71. Find the area of the triangle with vertices at the points:
(0, 0), (6, 0) and (4, 3)

72. Evaluate the following determinant:

$\begin{vmatrix}a+ib & c+id\\ -c+id & a-ib\end{vmatrix}$

73. Find the integral value of x, if $\begin{vmatrix}x^2 & x & 1\\ 0 & 2 & 1\\ 3 & 1 & 4\end{vmatrix}=28.$

74. Write the minors and cofactors of element of the first column of the following matrices and hence evaluate the determinant in case:

$A=\begin{vmatrix}-1 & 4\\ 2 & 3\end{vmatrix}$

75. If the points (3, -2), (x, 2), (8, 8) are collinear, find x using determinant.

76. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then for any natural number, find the value of Det (A^n).

77. If $A\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$, then show that |3A| = 27|A|.

78. Using determinants, find the equation of the line joining the points: (1, 2) and (3, 6)

79. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be sech that A^{-1} = kA, then find the value of k.

80. If $A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$, show that adj A = A.

81. Find the inverse of the following matrices:

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

82. If $\begin{bmatrix}1 & 0 & 0\\ 0 & y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ -1\\ z\end{bmatrix}=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$, find x, y and z.

83. show that $f(x)=\begin{cases}\frac{x-|x|}{2}, & \text{when } x\neq 0\\ 2, & \text{when } x=0\end{cases}$ is discontinuous at x = 0.

84. Discuss the continuity of the following functions at the indicated point:

$$f(x)=\begin{cases}(x-a)\sin\left(\frac{1}{x-a}\right) & x\neq a\\ 0, & x=a\end{cases} \text{ at } x=a$$

85. A function f(x) is defined as, $f(x)=\begin{cases}\frac{x^2-9}{x-3}, & \text{if } x\neq 3\\ 6, & \text{if } x=3\end{cases}$ show that f(x) is continuous that x = 3

86. In the following, determine the values of constants involved in the definition so that the given function is continuous:

$$f(x)=\begin{cases}\frac{\sin 2x}{5x}, & \text{if } x\neq 0\\ 3k, & \text{if } x=0\end{cases}$$

87. Differentiate:

$\tan(x^\circ+45^\circ)$

88. If $y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ \ldots\text{to }\infty}}}$ prove that $\frac{dy}{dx}=\frac{\sec^2 x}{2y-1}$

89. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\ \ldots\text{to }\infty}}}$, prove that $\frac{dy}{dx}=\frac{1}{2y-1}$

90. Find $\frac{dy}{dx}$ in the following cases:

$e^{x-y} = \log\left(\frac{x}{y}\right)$

91. Differentiate the following functions with respect to x:

$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right), \pi < x < \pi$

92. If $y = x \sin y$, prove that $\frac{dx}{dx} = \frac{\sin^2 y}{(1 - x\cos y)}$

93. If $y = 3e^{2x} + 2e^{3x}$ prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

94. Write the interval in which $f(x) = \sin x + \cos x, x \in \left[0, \frac{\pi}{2}\right]$ is increasing.

95. Show that $f(x) = \log \sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

96. Prove that the function f given by f(x) = x^3 - $3x^2$ + 4x is strictly increasing on R.

97. Prove that the function f given by f(x) = x - [x] is increasing in (0, 1).

98. Find the points o local maxima or local minima, if any, of the following functions, using the first derivatives test. Also, find the local maximum or local minimum values, as the case may be:

$f'(x) = x^3(2x-1)^3$

99. Find the points of local maxima or local minima and corresponding local maximum and local minimum values of the following functions. Also, find the points of inflection,

$f(x) = x\sqrt{1-x}, x \leq 1$

100. Show that $\frac{\log x}{x}$ has a minimum value at x = e.

➤ **Given Section consists of questions of 5 marks each.** **[200]**

101. Test whether the following relations R_1 are:

   i. Reflexive.
   ii. Symmetric.
   iii. Transitive.

R_1 on Q_0 defined by $(a, b) \in R_1 \Leftrightarrow a = \frac{1}{b}$.

102. Let R be a relation on the set A of ordered pair of integers defined by (x, y)R(u, v) if xv = yu. Show that R is an equivalence relation.

103. Let Z be the set of integers. Show that the relation R = {(a, b): a, b ∈ Z and a + b is even} is an equivalence relation on Z.

104. Let Z be the set of all integers and Z_0 be the set of all non-zero integers. Let a relation R on Z × Z_0 be defined as (a, b)R(c, d) ⇔ ad = bc for all (a, b), (c, d) ∈ Z × Z_0, Prove that R is an equivalence relation on Z × Z_0.

105. Let R be the relation over the set of all straight lines in a plane such that $l_1 R l_2 \Leftrightarrow l_1 \perp l_2$. Then, R is:

a. Symmetric.
b. Reflexive.
c. Transitive.
d. An equivalence relation.

106. Evaluate:

$\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)$

107. Prove the following results:

$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

108. Solve the following:

$\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

109. Prove the following results:

$2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$

110. Solve the following equation for x:

$\tan^{-1}\left(\frac{x-2}{x-1}\right)+\tan^{-1}\left(\frac{x+2}{x+1}\right)=\frac{\pi}{4}$

111. Show that $2\tan^{-1}x+\sin^{-1}\frac{2x}{1+x^2}$ is constant for $x \geq 1$, find that constant.

112. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3}$ and $\cos^{-1}x-\cos^{-1}y=\frac{\pi}{6}$, find the values of x and y.

113. Let $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$, Find A^T, B^T and verify that.

$(AB)^T=B^T+A^T$

114. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = 0$ use this to find A^4.

115. If $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$, show that AB = A and BA = B.

116. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

117. Find the value of x for which the matrix product

$\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}$ equal an identity matrix.

118. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, show that $A^2 - 7A + 10I_3 = 0$.

119. Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

120. Prove the following identities:

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$$

121. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A (adj A) = $|A| I_3$.

122. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

Show that

$[F(\alpha)]^{-1} = F(-\alpha)$

123. Solve the following systems of homogeneous linear equations by matrix method:

3x - y + 2z = 0

4x + 3y + 3z = 0

5x + 7y + 4z =0

124. Solve the following systems of homogeneous linear equations by matrix method:

x + y - z = 0

x - 2y + z = 0

3x + 6y - 5z = 0

125. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations:

2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y + 2z = -3

126. Find the points of discontinuity, if any of the following function:

$$f(x)=\begin{cases}\frac{\sin x}{x}, & \text{if } x<0\\ 2x+3, & x\geq 0\end{cases}$$

127. Discuss the continuity of the following functions at the indicated point:

$$f(x)=\begin{cases}\frac{1-x^n}{1-x}, & x\neq 1\\ n-1, & x=1\end{cases}\quad n \in \mathrm{N} \text{ at } x=1$$

128. In the following, determine the values of constants involved in the definition so that the given function is continuous:

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x<\frac{\pi}{2}\\ 3, & x=\frac{\pi}{2}\\ \frac{3\tan x}{2x-\pi}, & x>\frac{\pi}{2}\end{cases}$$

129. Discuss the continuity of the following functions at the indicated point:

$$f(x)=\begin{cases}|x-a|\sin(\frac{1}{x-a}), & \text{for } x\neq a\\ 0, & \text{for } x=a\end{cases}\text{ at } x=0$$

130. If the functions f(x), defined below is continuous at x = 0, find the value of k.

$$f(x)=\begin{cases}\frac{1-\cos 2x}{2x^2}, & x<0\\ k, & x=0\\ \frac{x}{|x|}, & x>0\end{cases}$$

131. Differentiate the following functions with respect to x:

$$\left(x+\frac{1}{x}\right)^x+x^{\left(1+\frac{1}{x}\right)}$$

132. If $x^y + y^x = (x + y)^{x+y}$, find $\frac{dy}{dx}$

133. Find $y=Ae^{-kt}\cos(pt+c)$ prove that $\frac{d^2y}{dt^2}+2k\frac{dy}{dt}+n^2y=0$, Where $n^2=p^2+k^2$.

134. Find A and B so that $y=A\sin 3x+B\cos 3x$ satisfy the equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=10\cos 3x$.

135. The surface area of a spherical bubble is increasing at the rate of $2cm^2/s$. When the radius of the bubble is 6cm, at what rate is the volume of the bubble increasing?

136. A balloon in the form of a right circular cone surmounted by a hemisphere, having a diametre equal to the height of the cone, is being inflated. How fast is its volume changing with respect to its total height h, when h = 9cm.

137. Show that $f(x)=\frac{1}{1+x^2}$ is decreases in the interval $[0,\infty)$ and increases in the interval $(-\infty,0]$.

138. Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 5 + 36x + 3x^2 - 2x^3$

139. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12cm is 16cm.

140. A wire of length 20m is to be cut into two pieces. One of the pieces will be bent into shape of a square and the other into shape of an equilateral triangle. Where the we should be cut so that the sum of the areas of the square and triangle is minimum ?

➤ Case study based questions [40]

141. Consider the mapping f: A → B is defined by f(x) = x - 1 such that f is a bijection. Based on the above information, answer the following questions.

   i. Domain of f is:
      a. R - {2}
      b. R
      c. R - {1, 2}
      d. R - {0}

   ii. Range of f is:
      a. R
      b. R - {2}
      c. R - {0}
      d. R - {1, 2}

   iii. If g: R - {2} → R - {1} is defined by g(x) = 2f(x) - 1, then g(x) in terms of x is:
      a. $\frac{x+2}{x}$
      b. $\frac{x+1}{x-2}$
      c. $\frac{x-2}{x}$
      d. $\frac{x}{x-2}$

   iv. The function g defined above, is:
      a. One-one
      b. Many-one
      c. into
      d. None of these

   v. A function f(x) is said to be one-one iff.
      a. $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$
      b. $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$
      c. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
      d. None of these

142. A relation R on a set A is said to be an equivalence relation on A iff it is:

   I. Reflexive i.e., $(a, a) \in R \ \forall a \in A$.

II. Symmetric i.e., $(a, b) \in R \Rightarrow (b, a) \in R \ \forall\ a, b \in A$.

III. Transitive i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall\ a, b, c \in A$.

Based on the above information, answer the following questions.

i. If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then R is:
   a. Reflexive
   b. Symmetric
   c. Transitive
   d. Equivalence

ii. If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then R is:
   a. Reflexive
   b. Symmetric
   c. Transitive
   d. Equivalence

iii. If the relation R on the set N of all natural numbers defined as R = {(x, y): y = x + 5 and x < 4}, then R is:
   a. Reflexive
   b. Symmetric
   c. Transitive
   d. Equivalence

iv. If the relation R on the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y): 3x - y = O}, then R is:
   a. Reflexive
   b. Symmetric
   c. Transitive
   d. Equivalence

v. If the relation R on the set A = {I, 2, 3} defined as R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}, then R is:
   a. Reflexive only
   b. Symmetric only
   c. Transitive only
   d. Equivalence

143. Three friends Ravi, Raju and Rohit were doing buying and selling of stationery items in a market. The price of per dozen of pen, notebooks and toys are Rupees x, y and z respectively.

Ravi purchases 4 dozen of notebooks and sells 2 dozen of pens and 5 dozen of toys. Raju purchases 2 dozen of toy and sells 3 dozen of pens and 1 dozen of notebooks. Rohit purchases one dozen of pens and sells 3 dozen of notebooks and one dozen of toys.

In the process, Ravi, Raju and Rohit earn ₹1500, ₹100 and ₹ 400 respectively.

(i) Write the above information in terms of matrix Algebra.

(ii) What is the total price of one dozen of pens and one dozen of notebooks?

(iii) What is the sale amount of Ravi?

OR

What is the amount of purchases and sales made by all three friends?

144. Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.

(i) Write the matrix summarizing sales data of 2019 and 2020.

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

145. A trust fund has ₹35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.


(i) Represent the given information in matrix algebra.

(ii) If ₹ 15000 is invested in bond X, then find total amount of interest received on both bonds?

(iii) If the trust fund obtains an annual total interest of ₹ 3200 , then find the investment in two bonds.

OR

If the amount of interest given to old age home is ₹500, then find the amount of investment in bond Y.

146. A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is ₹$100/\mathrm{m}^2$.

(i) If $r \mathrm{cm}$ be the radius and $h \mathrm{cm}$ be the height of the cylindrical tin can, then express the surface area as a function of radius (r)

(ii) Find the radius of the can that will minimize the cost of tin used for making can?

(iii) Find the height that will minimize the cost of tin used for making can ?

OR

Find the minimum cost of material used to manufacture the tin can.

147. A function f(x) is said to be continuous in an open interval (a, b), if it is continuous at every point in this interval.

A function f(x) is said to be continuous in the closed interval [a, b), if f(x) is continuous in (a, b) and $\lim_{x\to 0} f(a+h) = f(a)$ and $\lim_{x\to 0} f(b-h) = f(b)$ If function $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , x < 0 \\ c & , x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & , x > 0 \end{cases}$ is continuous at x = 0, then answer the following questions.

i. The value of a is:
   a. $-\frac{3}{2}$
   b. 0
   c. $\frac{1}{2}$
   d. $-\frac{1}{2}$

ii. The value of b is:
   a. 1
   b. -1
   c. 0
   d. Any real number.

iii. The value of c is:
   a. 1
   b. $\frac{1}{2}$
   c. -1
   d. $-\frac{1}{2}$

iv. The value of a + c is:
   a. 1
   b. 0
   c. -1
   d. -2

v. The value of c - a is:
   a. 1
   b. 0
   c. -1
   d. 2

148. Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$, where both u(x) and v(x) are differentiable functions and f and u need to be positive functions.

Let function $y = f(x) = (u(x))^{v(x)}$, then $y' = y\left[\frac{v(x)}{u(x)}u'(x) + v'(x) \cdot \log[u(x)]\right]$ On the basis of above information, answer the following questions.

i. Differentiate x^x w.r.t. x.

a. $x^x(1+\log x)$

b. $x^x(1-\log x)$

c. $-x^x(1+\log x)$

d. $x^x \log x$

ii. Differentiate $x^x + a^x + x^a + a^a$ w.r.t. x.

a. $(1+\log x)+(a^x \log a + ax^{a-1})$

b. $x^x(1+\log x)+\log a + ax^{a-1}$

c. $x^x(1+\log x)+x^a \log x + ax^{a-1}$

d. $x^x(1+\log x)+a^x \log a + ax^{a-1}$

iii. If $x = e^{\frac{x}{y}}$, then find $\frac{dy}{dx}$.

a. $-\frac{(x+y)}{x \log x}$

b. $-\frac{(x-y)}{x \log x}$

c. $\frac{(x+y)}{x \log x}$

d. $\frac{x-y}{x \log x}$

iv. If $y = (2 - x)^3(3 + 2x)^5$, then find $\frac{dy}{dx}$.

a. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}-\frac{8}{2-x}\right]$

b. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}+\frac{3}{2-x}\right]$

c. $(2-x)^3(3+2x)^5\left[\frac{10}{3+2x}-\frac{3}{2-x}\right]$

d. $(2-x)^3(3+2x)^5 \cdot \left[\frac{10}{3+2x}+\frac{3}{2-x}\right]$

v. If $y = x^x \cdot e^{(2x+5)}$, then find $\frac{dy}{dx}$.

a. $x^x e^{2x+5}$

b. $x^x e^{2x+5}(3-\log x)$

c. $x^x e^{2x+5}(1-\log x)$

d. $x^x e^{2x+5} \cdot (3+\log x)$

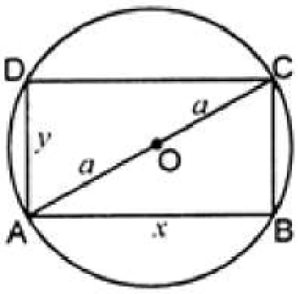
149. Read the following passage and answer the questions given below: elation between the height of the plant (y' in cm) with respect to its exposure to is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where ' x ' is the number of days exposed to the sunlight, for $x \leq 3$

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days?

What will be the height of the plant after 2 days?

150. A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

(iii) Check for maximum or minimum value of perimeter at critical point.

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

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